

# Bound on the variation in the fine structure constant implied by Oklo data

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Dynamical models of dark energy can imply that the fine structure constant  $\alpha$  varies over cosmological time scales. Data on shifts in resonance energies  $E_r$  from the Oklo natural fission reactor have been used to place restrictive bounds on the change in  $\alpha$  over the last 1.8 billion years. We review the uncertainties in these analyses, focussing on corrections to the standard estimate of  $k_\alpha = \alpha dE_r/d\alpha$  due to Damour and Dyson. Guided, in part, by the best practice for assessing systematic errors in theoretical estimates spelt out by Dobaczewski *et al.* [in J. Phys. G: Nucl. Part. Phys. **41**, 074001 (2014)], we compute these corrections in a variety of models tuned to reproduce existing nuclear data. Although the net correction is uncertain to within a factor of 2 or 3, it constitutes at most no more than 25% of the Damour-Dyson estimate of  $k_\alpha$ . Making similar allowances for the uncertainties in the modeling of the operation of the Oklo reactors, we conclude that the relative change in  $\alpha$  since the Oklo reactors were last active (redshift  $z \simeq 0.14$ ) is less than  $\sim 10$  parts per billion. To illustrate the utility of this bound at low- $z$ , we consider its implications for the string theory-inspired runaway dilaton model of Damour, Piazza and Veneziano.

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# 1 Introduction

In the August, 2015 issue of *Physics Today*, Frank Wilczek, reviewing a new anthology of Freeman Dyson’s papers, writes that “Dyson’s paper, with Thibault Damour, placing empirical limits on the possible time variation of the fine-structure and other fundamental ‘constants’ is a gem within *Birds and Frogs*.” Since its appearance, Damour and Dyson’s paper [1] has provided *the* basis for relating Oklo data to the fine-structure constant  $\alpha$ , but, in a recent paper [2], we have reconsidered the foundations of their method. This contribution to DPF 2015 is a digest of some of the principal ideas and results of our paper, but it is a more visual presentation, with diagrams generated specifically for DPF 2015 (Ref. [2] does not contain any figures!). In addition, we briefly take up a topic not discussed in Ref. [2], namely the implications of our findings for recent predictions [3] of the redshift dependence of  $\alpha$  in the runaway dilaton model [4].

## 2 What is Oklo and why is it of interest?

There is an extensive body of work on the Oklo phenomenon, the inactive natural nuclear fission reactors discovered in 1972 by the CEA (see Ref. [5] for a recent review of the literature). Of most interest to us is the fact that these reactors were last active about 1.8 to 2 billion years ago. Thus, Oklo geochemical data provide us with a record of nuclear processes, specifically neutron capture by complex nuclei, at redshift  $z \simeq 0.14$ .

As first recognised by Alexander I. Shlyakhter [6], there would have been Oklo capture cross-sections which were dominated by the properties of a single compound-nucleus resonance and any difference in such a cross-section from its present-day value can be translated into a change  $\Delta_r$  in the resonance energy  $E_r$  over the intervening time. Shlyakhter indicated through simple back-of-envelope estimates how a bound on  $\Delta_r$  could be used to constrain the change in the dimensionless strengths of the strong, weak and electromagnetic interactions over the same time interval.

In work to improve on Shlyakhter’s estimates, the reaction  $n + {}^{149}\text{Sm}$  has received most attention, primarily because the cross section for thermal neutron capture is huge (with a resonance just above threshold at 97.3 meV), offering the best possibility for the determination of  $\Delta_r$  (a small change in  $E_r$  manifests itself in a dramatic change in the capture rate). The information on  $\Delta_r$  extracted also depends on the modelling of the neutron spectrum within the Oklo reactors. Beginning with Ref. [1], there have been several independent studies of this issue [7, 8] and more work would be appreciated! Nevertheless, despite the many uncertainties, the 3 tightest bounds on  $\Delta_r$  inferred from Sm data all agree to within a factor of 2 with the result of Ref. [8], which we, henceforth, adopt:  $\Delta_r = 7.2 \pm 9.4 \text{ meV}$ .

### 3 How to extract a bound on $\Delta\alpha \equiv \alpha_{\text{Oklo}} - \alpha_{\text{now}}$

The Damour-Dyson method is based on two inequalities. First, there is the plausible assumption that the *net* change in  $\Delta_r$  exceeds the change due to  $\Delta\alpha$  alone, or\*

$$|\Delta_r| \geq |k_\alpha| \frac{|\Delta\alpha|}{\alpha_{\text{now}}}, \quad (1)$$

where  $k_\alpha \equiv dE_r/d\ln\alpha$ . It follows that a lower bound to  $|k_\alpha|$  is enough to set an upper bound on  $|\Delta\alpha|$ . Via the Hellmann-Feynman theorem,  $k_\alpha$  can be related to a difference in nuclear coulomb energies [5], and, by discarding the small exchange contribution to nuclear coulomb energies and adroit use of Green's second identity, a second inequality can be derived [1], namely, the upper bound

$$k_\alpha \leq \int V_\star(\rho_{150^\star} - \rho_{149})d^3r, \quad (2)$$

where  $V_\star$  is the electrostatic potential of the excited compound nucleus  $^{150}\text{Sm}$ ,  $\rho_{150^\star}$  is its charge density, and  $\rho_{149}$  is the ground state charge density of  $^{149}\text{Sm}$ . The righthand side of Eq. (2) proves to be negative, and so its magnitude is a candidate for the desired lower bound to  $|k_\alpha|$ .

In the evaluation of the righthand side of Eq. (2), Damour and Dyson make two uncontrolled approximations. They take the charge distribution of the  $^{150}\text{Sm}$  compound nucleus to be a sphere of uniform density, and set

$$V_\star = \frac{Ze}{2R^3}(3R^2 - r^2) \quad (3)$$

for *all* radial distances  $r$  from the center of the sphere ( $Ze$  is the charge within the sphere and  $R$  is its radius), meaning that, for any choices of charge densities  $\rho_1$  and  $\rho_2$  (normalised to  $Ze$ ),

$$\int V_\star(\rho_1 - \rho_2)d^3r = -Ze (\langle r^2 \rangle_1 - \langle r^2 \rangle_2),$$

where  $\langle r^2 \rangle_i$  denotes the mean square radial moment of the charge density  $\rho_i$ . They further replace the mean square radial moment for the compound nucleus by its value

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\*A more complete treatment [2] of the relation between  $\Delta_r$  and  $\Delta\alpha$  includes the effect of a change in the ratio  $X_q$  of the average light quark mass  $m_q = \frac{1}{2}(m_u + m_d)$  to the QCD mass scale  $\Lambda$ ; Eq. (1) is replaced by

$$|\Delta_r| = \left| k_\alpha \frac{\Delta\alpha}{\alpha_{\text{now}}} + k_q \frac{\Delta X}{X_{\text{now}}} \right| \geq |\gamma\nu - 1| |k_\alpha| \frac{|\Delta\alpha|}{\alpha_{\text{now}}},$$

where  $\gamma \equiv |k_q/k_\alpha|$  and  $\nu \equiv |\Delta X/X_{\text{now}}/\Delta\alpha/\alpha_{\text{now}}|$ : as estimates [2, 9] of  $k_q$  and  $k_\alpha$  imply that  $|\gamma| \gtrsim 4$ , a sufficient condition for the validity of Eq. (1) is that the BSM input  $\nu > \frac{1}{2}$ . This would appear to be a weak restriction, respected by all existing phenomenological determinations of  $\nu$ .

for the ground state of  $^{150}\text{Sm}$  ( $\langle r^2 \rangle_{150^*} \rightarrow \langle r^2 \rangle_{150}$ ) to obtain the final result

$$k_\alpha \leq k^{DD} \equiv -\frac{(Ze)^2}{2R^3}(\langle r^2 \rangle_{150} - \langle r^2 \rangle_{149}). \quad (4)$$

The seductive appeal of the Damour-Dyson estimate  $k^{DD}$  of a bound on  $k_\alpha$  is that it can be calculated by reference to experimental data (on isotope shifts and equivalent rms charge radii) alone. It is unfortunate then that, in computing  $k^{DD}$ , Damour and Dyson choose an unphysically large value of  $R$  (8.11 fm instead of  $6.50 \pm 0.01$  fm from tabulations of charge radii). Our preferred value of  $k^{DD}$  is  $-2.51 \pm 0.20$  MeV as opposed to the significantly smaller value of  $-1.1 \pm 0.1$  MeV advocated by Damour and Dyson.

We can identify three nuclear physics corrections to the result in Eq. (4): an excitation correction, an external coulomb correction, and a deformation correction. The excitation and external coulomb corrections compensate, respectively, for the replacement of  $\langle r^2 \rangle_{150^*}$  by  $\langle r^2 \rangle_{150}$  and for the use throughout all space of Eq. (3), a result appropriate to the *inside* of a sphere of uniform volume charge density. The deformation correction accommodates the fact that both the ground state of  $^{149}\text{Sm}$  and the compound nucleus state of  $^{150}\text{Sm}$  excited have prolate deformations.

We need more realistic charge densities to compute these corrections. We have employed deformed Fermi profiles fitted to the output of Hartree-Fock+BCS calculations and developed a method for including the increase in surface diffuseness with excitation energy. Our results for four different models are plotted in Fig. 1 for a reasonable range of  $\Delta\beta = \beta_\star - \beta_{\text{gs}}$ , where  $\beta_\star$  and  $\beta_{\text{gs}}$  are the quadrupole deformation parameters for the compound nucleus state excited in  $^{150}\text{Sm}$  and the ground state of  $^{150}\text{Sm}$ , respectively. There are significant cancellations between the excitation and deformation corrections with the *net* correction  $k^{\text{corr}}$  never exceeding 25% of  $k^{DD}$ .

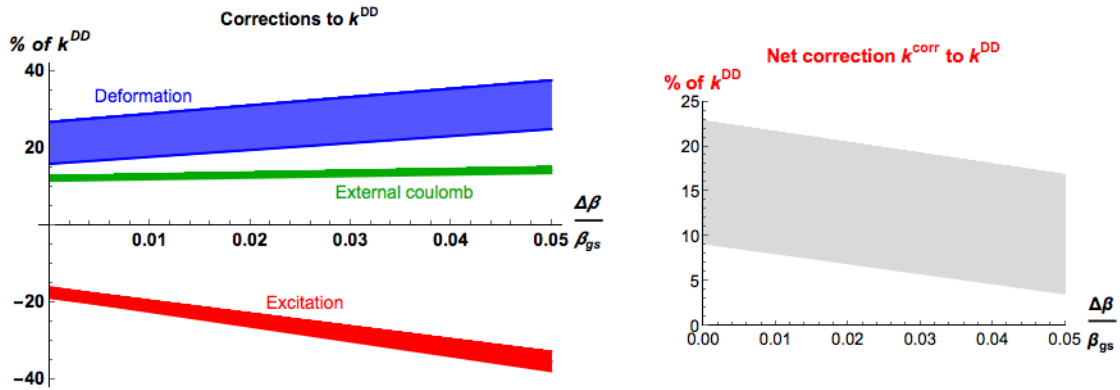


Figure 1: Individual corrections to  $k^{DD}$  and the *net* correction  $k^{\text{corr}}$ . The bands plotted delineate the range of values obtained with the four different models of densities considered.

On the basis [10] of the mean and the scatter of estimates of  $k^{\text{corr}}$ , we conclude that  $k^{\text{corr}} = 0.33 \pm 0.16 \text{ MeV}$  or, combining the errors of  $k^{DD}$  and  $k^{\text{corr}}$  in quadrature,

$$k_\alpha \leq k_{\text{Bd}} \equiv k^{DD} + k^{\text{corr}} = -2.18 \pm 0.26 \text{ MeV}.$$

Despite an uncertainty in  $k^{\text{corr}}$  of about 50%, the error in the bound on  $k_\alpha$  is only a little over 10% or so.

From Eqs. (1) and (2),  $|\Delta\alpha|/\alpha_{\text{now}}$  is bounded by  $|\Delta_r|/(-k_{\text{Bd}})$ . If we interpret  $\zeta = -\Delta_r/k_{\text{Bd}}$  as the ratio of two gaussian random variables of known mean and standard deviation, then it turns out that  $\zeta$  itself is, to an excellent approximation, also a gaussian random variable. Its probability density is shown in Fig. 2, along with, for the sake of illustration, the area (in pink) associated with the 68% C.L. upper bound on  $|\zeta|$ . Using the distribution of  $\zeta$ , we determine the bound of

$$\frac{|\Delta\alpha|}{\alpha_{\text{now}}} < 1.1 \times 10^{-8} \quad (95\% \text{ C.L.}). \quad (5)$$

## 4 Implications for dark cosmology

Figure 3 depicts the relation between the redshift dependence of  $\alpha$  and the current “speed”  $\Phi'_0$  of the dilaton field in the model of Ref. [4] for the updated parameter values of Ref. [3]. Setting  $z = 0.14$  and  $|\alpha_{\text{had}}| = 10^{-4}$  (from Ref. [3]) in

$$\frac{|\Delta\alpha|}{\alpha} \simeq \frac{|\alpha_{\text{had}}|}{40} |\Phi'_0| \ln(1+z),$$

where  $\alpha_{\text{had}}$  denotes the coupling of the dilaton to hadronic matter, our bound on  $|\Delta\alpha|$  in Eq. (5) implies that  $|\Phi'_0| \lesssim 0.03$ . Inspection of Fig. 3 suggests that, for this range of dilation speeds and the redshifts shown, the difference in the  $z$ -dependence of  $\alpha$  for  $\Lambda\text{CDM}$  and for the dilaton model will be undetectable.

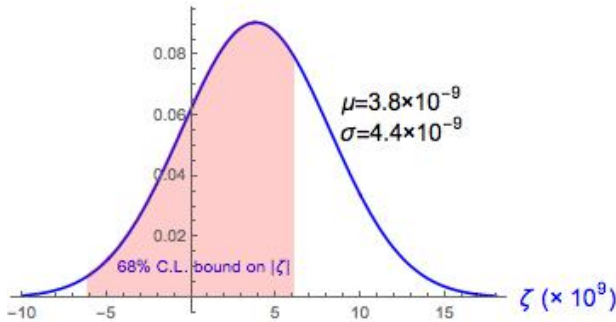


Figure 2: Gaussian distribution of  $\zeta \equiv -\frac{\Delta_r}{k_{\text{Bd}}}$ .

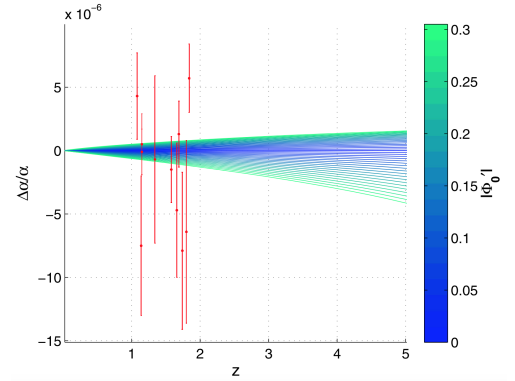


Figure 3:  $\Delta\alpha(z)$  in the model of Ref. [4] (taken from Fig. 2 of Ref. [3]).

## 5 Conclusion

There are 3 principal conclusions: i) despite suggestions to the opposite in the literature, the (revised) Damour-Dyson estimate of  $k_\alpha$  is reliable for order of magnitude purposes; ii) at 95% C.L.,  $|\alpha_{\text{Oklo}} - \alpha_{\text{now}}|/\alpha_{\text{now}} < 1.1 \times 10^{-8}$ , and; (iii) our bound on  $|\alpha_{\text{Oklo}} - \alpha_{\text{now}}|$  implies that, for redshifts  $z < 5$ , the behaviour of  $\alpha$  in the runaway dilaton model and in  $\Lambda$ CDM is almost indistinguishable.

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